THEORETICAL AND EXPERIMENTAL INVESTIGATION OF ASYMMETRIC COPLANAR WAVEGUIDES

Victor Fouad Hanna and Dominique Thebault

Centre National d'Etudes des Télécommunications (CNET)
Centre Paris B, Division ETR
92131 Issy Les Moulineaux - France

Abstract

Conformal mapping techniques are used to obtain analytic closed-form expressions for the characteristic impedance and the relative effective dielectric constant of asymmetric coplanar waveguide with infinite or finite dielectric thickness. The line asymmetry leads to a decrease of its characteristic impedance and to an increase of its relative effective dielectric constant. Six asymmetric coplanar waveguides are realised and their characteristic impedances are measured using time domain reflectometry techniques.

Introduction

Recently, symmetric coplanar waveguides "CPW" (1) have been the subject of growing interest as they have presented a solution to technical and technological problems encountered in the design of microstrip and slot transmission lines due to their easy adaptation to external shunt element connections as well as monolithic circuits. Here, we study the asymmetric CPW shown in Fig. 1a ($d_1 \neq d_2$). The study of such a line is important as it allows one to evaluate the actual characteristics of a CPW normally designed to be symmetric, but the fabrication of which is imperfect.

During the whole analysis, we assume the ground planes to be infinitely wide and the strips to have negligible thickness. Also we assume that the air-dielectric interfaces can be dealt with as though perfect magnetic walls were present in them

Infinite Dielectric Thickness Asymmetric CPW

We (2) have treated this case in details using conformal mapping techniques. We give here only the results. The characteristic line impedance Z_0 and the total line capacitance per unit length C can be written in the form

$$Z_0 = \frac{30\pi}{\sqrt{(1+\epsilon_r)/2}} \cdot \frac{K'(k_1)}{K(k_1)} \qquad (1)$$

$$C = 2\epsilon_0(1+\epsilon_r)\ [K(k_1/K'(k_1)] \qquad (2)$$

K(k) is the complete elliptical integral of the first kind and $K'(k) = K(k')$, $k' = (1-k^2)^{\frac{1}{2}}$

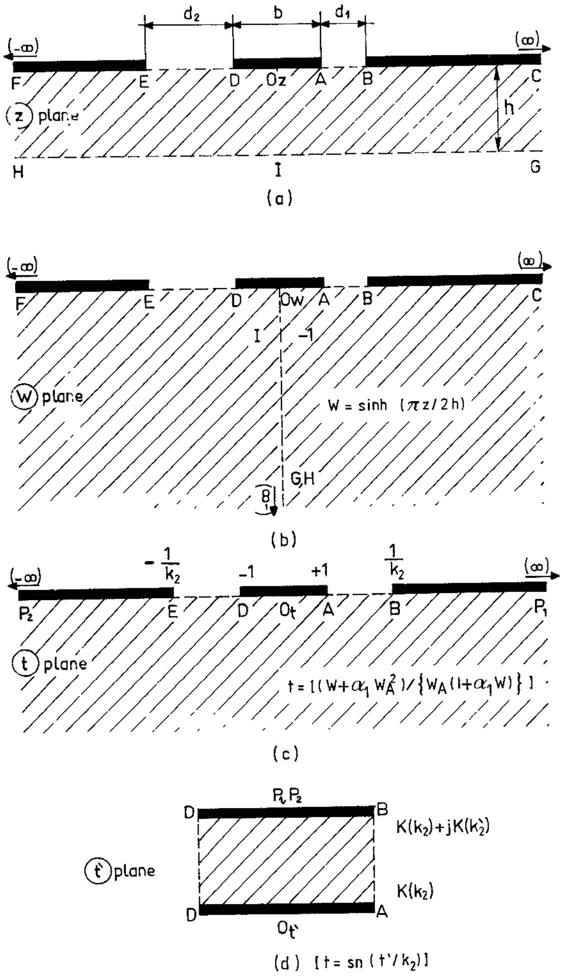

Fig.1: Conformal transformations for calculating C_2

and

$$k_1 = \frac{\frac{1}{2}b\left[1+\alpha\left(\frac{1}{2}b+d_1\right)\right]}{\frac{1}{2}b+d_1+\alpha\left(\frac{1}{2}b\right)^2} \qquad (3)$$

$$\alpha = \frac{d_1 d_2 + \frac{1}{2}b(d_1+d_2) \pm \left[d_1 d_2 (b+d_1)(b+d_2)\right]^{\frac{1}{2}}}{\left(\frac{1}{2}b\right)^2 (d_1-d_2)} \qquad (4)$$

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Finite Dielectric Thickness Asymmetric CPW

In this case, we assume that the total line capacitance per unit length is equal to the sum of the line capacitance per unit length in free space when the dielectric is replaced by air C_1 and the line capacitance per unit length C_2 obtained when assuming that the electric field is concentrated in a dielectric of thickness h and relative permittivity ($\epsilon_r - 1$). This assumption has shown an excellent accuracy in the case of symmetric CPW (3). So C_1 can be obtained from equation (2) by putting ϵ_r = 1, ie.

$$C_1 = 4\epsilon_0 \frac{K(k_1)}{K'(k_1)} \tag{5}$$

The line capacitance C_2 can be computed through a sequence of three intermediate conformal mappings (see Figs. 1b, 1c and 1d).

$$W = \sinh\left(\frac{\pi z}{2h}\right) \tag{6}$$

$$t = \frac{W + \alpha_1 W_A^2}{W_A(1+\alpha_1 W)} \tag{7}$$

$$t = \mathrm{sn}(t', k_2) \tag{8}$$

where sn(t,k) is the sine elliptic function.

We have to notice that the conformal transformation of equation (7) is used to transform the asymmetrical boundary value problem of the W plane into a symmetrical one in the t plane. Here, α_1 and k_2 can be obtained from the symmetry condition

$$W_B = -W_E = \frac{1}{k_2} \tag{9}$$

then, C_2 can be given by the relations

$$C_2 = 2\epsilon_0(\epsilon_r - 1)\frac{K(k_2)}{K'(k_2)} \tag{10}$$

Finally, the relative effective dielectric constant ϵ_{eff} and the characteristic impedance Z_0 of the asymmetric CPW can be written as

$$\epsilon_{eff} = \frac{C_1 + C_2}{C_1} = 1 + \frac{1}{2}(\epsilon_r - 1)\frac{K(k_2)}{K'(k_2)} \cdot \frac{K'(k_1)}{K(k_1)} \tag{11}$$

$$Z_0 = \frac{30\pi}{(\epsilon_{eff})^{\frac{1}{2}}} \frac{K'(k_1)}{K(k_1)} \tag{12}$$

where

$$k_2 = \frac{W_A(1+\alpha_1 W_B)}{W_B + \alpha_1 W_A^2} \tag{13}$$

$$\left.\begin{aligned} W_A &= \sinh\left(\frac{\pi b}{4h}\right),\ W_B = \sinh\left[\frac{\pi}{2h}\left(\frac{b}{2} + d_1\right)\right] \\ W_E &= -\sinh\left[\frac{\pi}{2h}\left(\frac{b}{2} + d_2\right)\right] \end{aligned}\right\} \tag{14}$$

$$\alpha_1 = (W_B + W_E)^{-1}\left[-1 - \frac{W_B W_E}{W_A^2} \pm \left\{\left(\frac{W_B^2}{W_A^2} - 1\right)\left(\frac{W_E^2}{W_A^2} - 1\right)\right\}^{\frac{1}{2}}\right] \quad \ldots\ldots (15)$$

Hence, the line parameters can be calculated from (11) and (12) using the simple formulas of Hilberg (4) for the ratio K(k)/K'(k)

Examples of design curves are given in Figs. 2, 3 and 4

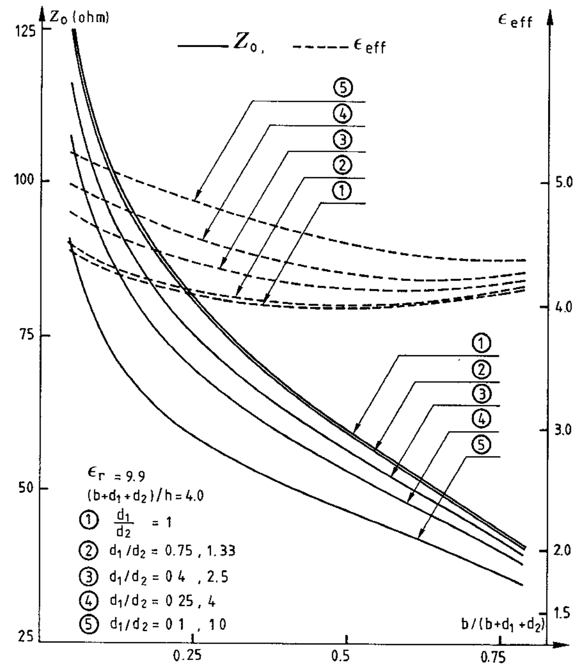

Fig. 2: Variation of Z_0 and ϵ_{eff} as a function of b/(b+d_1+d_2) for the shown five asymmetrical CPWs

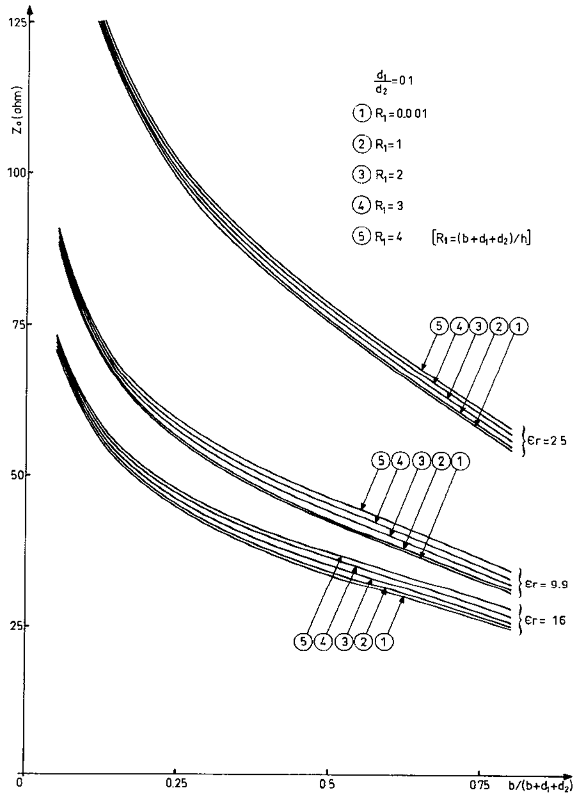

Fig. 3: Variation of Zo as a function of b/b+d_1+d_2) for the shown asymmetrical CPW parameters

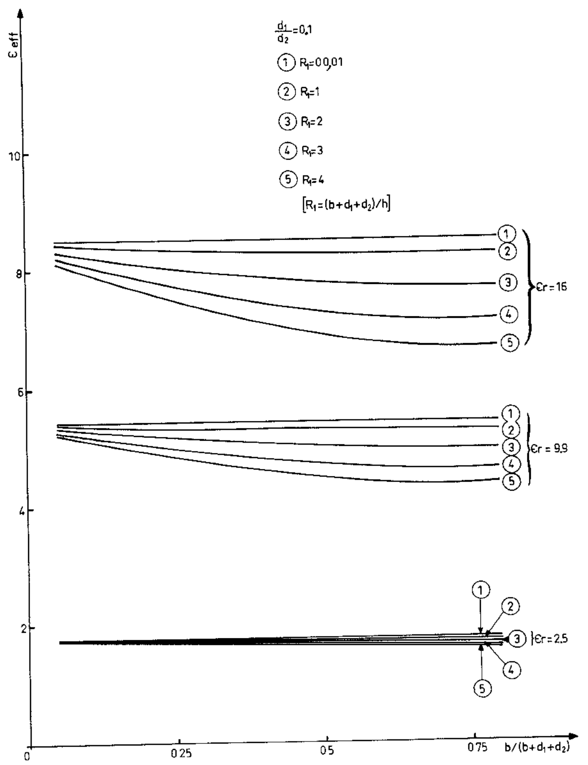

Fig.4: Variation of ϵ_{eff} as a function of $b/b+d_1+d_2$) for the shown asymmetrical CPW parameters

From our calculations, it can be concluded that, for a given shape ratio $b/(b + d_1 + d_2)$, the line asymmetry leads to a decrease of its characteristic impedance and to an increase of its relative effective dielectric constant. However, these variations are not significant for asymmetry factors up to 25 % (the asymmetry factor s can be defined as $s = 1 - (d_1/d_2)$ if $d_1 \leqslant d_2$ or $s = 1 - (d_2/d_1)$ if $d_2 \leqslant d_1$). From Figs. 3 and 4, it can be seen that the line characteristic impedance and its relative effective dielectric constant differ by less than 2 and 4 %, respectively, when the thickness of the substrate is reduced from infinity to $(b+d_1+d_2)$ and by less than 10 and 20 % respectively when h is reduced to one third of $(b + d_1 + d_2)$. So it is important to not neglect the effect of the finite dielectric thickness when the condition $h \geqslant (b+d_1+d_2)$ is not satisfied.

Experimental Results

Six asymmetric CPWs have been realised. Each was fabricated on an alumina substrate (ϵ_r = 9.9 and h = 0.635 mm) metalised with gold of thickness 4 μm. The characteristic impedances of these lines are measured using time domain refelctomtry techniques and they are calculated firstly using equation (1) ($h = \infty$) and secondly using equation (12) (h = finite value). The line dimensions as well as the theoretical and experimental results are given in Table 1. These experimental results are in

Table 1. Parameters of the six realised asymmetrical CPWs

Line Number	d_1 (μm)	d_2 (μm)	b (μm)	$Z_0\|_{h=\infty}$ (ohm)	$Z_0\|_{h = finite}$ (ohm)	$Z_0\|_{measured}$ (ohm)
1	123	1 060	747	48.63	51.78	51.5
2	257	991	737	55.13	59.88	57.5
3	356	843	735	57.33	62.96	61.1
4	196	1 756	1 250	48.18	53.53	52
5	406	1 548	1 248	54.02	62.38	62.4
6	575	1 386	1 244	56.69	67.17	66.3

a very good agreement with the theoretical ones and show the importance of not neglecting the substrate thickness when the condition $h \geqslant (b+d_1+d_2)$ is not fulfilled.

Conclusion

Analytic closed-form expressions for the characteristic impedance and the relative effective dielectric constant for asymmetric CPW with finite or infinite substrate thickness are obtained using conformal mapping techniques. Their exactitude has been verified experimentally.

Our results show that the CPW asymmetry leads to a decrease of the average performance range of its characteristic impedance and to an increase of its relative effective dielectric constant.

References

(1) C.P. Wen, "Coplanar Waveguide : a Surface Strip Transmission Line", IEEE Trans. on MTT, Vol. MTT-17, pp. 1087-1090, (1969).

(2) V. Fouad Hanna and D. Thebault, "Analysis of Asymmetrical Coplanar Waveguides", Int. J. Electron., Vol. 50, pp. 221-224, (1981).

(3) V. Fouad Hanna and C. Veyres, "Extension of the Application of Conformal Mapping Techniques to Coplanar Lines with Finite Dimensions", Int. J. Electron., Vol. 48, pp. 47-56, (1980).

(4) W. Hilberg, "From Approximations to Exact Relations for Characteristic Impedances", IEEE Trans. on MTT, Vol. MTT-17, pp. 259-265, (1969).